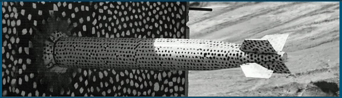
Applying genetic programming symbolic regression to solid mechanics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Abstract

Analytic relations describing the mechanics of solids are valuable.

- Inherent interpretability builds trust and understanding.
- Efficiently and practically implemented into existing workflows.

Generally, is it challenging to obtain realistic analytic relations.

- Severely limiting assumptions are required for traditional derivation.
- Microstructural simulations are expensive and hard to describe.
- Many machine learning approaches do not provide analytic relations.

Genetic programming symbolic regression for solid mechanics.

- Machine learning approach that produces analytic relations describing training data.
- Interpretable, data-driven, and physics-based models without limiting assumptions.

Motivation

Gurson model for porous plasticity [1].

- Implicit relation for yield surface.
- Focus of our collaborators at Utah & NASA.

Cocks-Ashby model for porous damage [2].

- Explicit damage evolution law.
- Focus of ongoing effort at Sandia.

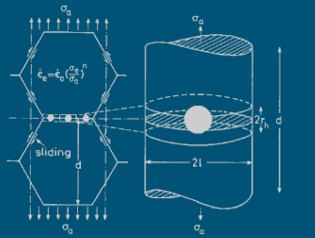
Some common limiting assumptions:

- Perfect plasticity or power-law creep.
- Self-similar growth of spherical pores.
- Negligible interactions of pores.

Obtain analytic models with less assumptions?

$$\Phi = \left(\frac{\sigma_e}{\sigma_y}\right) + 2\phi \cosh\left(\frac{3\sigma_h}{2\sigma_y}\right) - 1 - \phi^2$$

$$\dot{\phi} = \sqrt{\frac{3}{2}}\,\dot{\varepsilon}_p \,\frac{1-(1-\phi)^{n+1}}{(1-\phi)^n}\,\sinh\left[\frac{2(2n-1)}{2n+1}\,\frac{\sigma_h}{\sigma_e}\right]$$



Voids growing on grain boundary in power-law creeping solid

Approach

Genetic programming:

- Model evolution based on some notion of fitness.

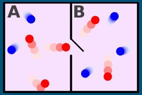
Symbolic regression:

- Fits analytic mathematical expressions to data.

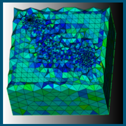
Bingo:

- Python/C++ package for GPSR [3].
- Developed by collaborators at Utah and NASA.

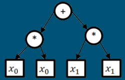
github.com/nasa/bingo



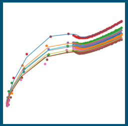
Identify physics



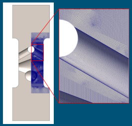
Generate data



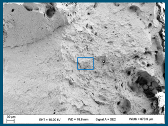
Apply GPSR



Obtain model



Apply model



Refine and repeat

Approach

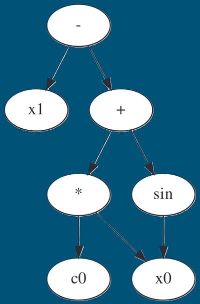
Genetic programming:

- Individuals as acyclic graphs.
- Crossovers, mutations.
- Selection, archipelagos.
- Hall of fame as Pareto front.

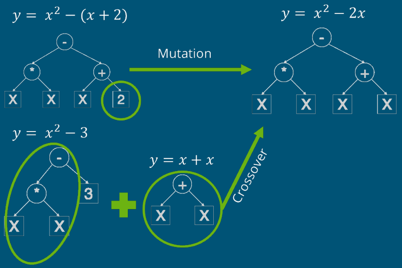
Symbolic regression:

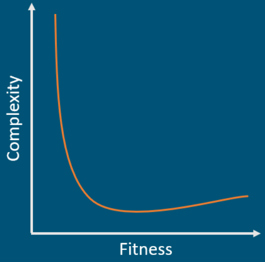
- Mathematical evaluation.
- Individual optimizations.
- Explicit (damage evolution).
- Implicit (yield surface).

$$f(\mathbf{x}) = f_0, \quad \frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \cdot \dot{\mathbf{x}} = 0$$



$$x_1 - (c_0 x_0 + \sin x_0)$$





Example

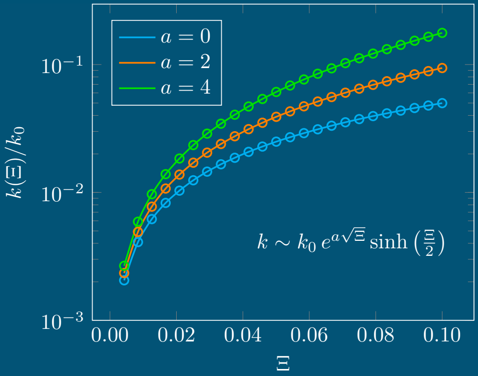
Simple model for slow crack growth [4].

```
import numpy as np
x = np.logspace(-3, -1, 25)
y = np.exp(np.sqrt(x))*np.sinh(x/2)

from bingo.symbolic_regression import ExplicitTrainingData
data = ExplicitTrainingData(x, y)

from bingo.symbolic_regression import ComponentGenerator
component_generator = ComponentGenerator()
component_generator.add_operator("+")
component_generator.add_operator("-")
component_generator.add_operator("*")
component_generator.add_operator("/")
component_generator.add_operator("exp")

while island.get_best_fitness() > tolerance:
    island.evolve(1)
```



Summary:

GPSR is relatively easy to apply and is usually capable of recovering existing analytic models.

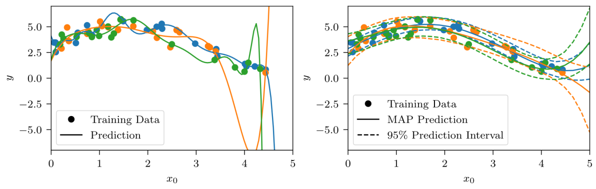
Plasticity

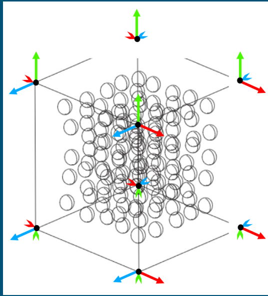
Implicit GPSR for yield surfaces:

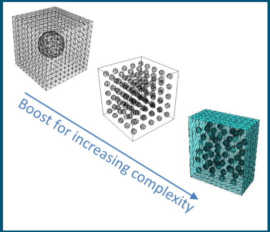
- Recover Von Mises from trivially generated data [5].
- Recover Gurson from FEA with consistent assumptions [6].
- Obtain new models from FEA with relaxed assumptions [5, 6].

Recent exciting progress:

- Boosting and seeding [6], theoretical constraints [7].
- Bayesian inference for uncertainty, reduced over-fitting [8].







Damage

AM 316L SS material.

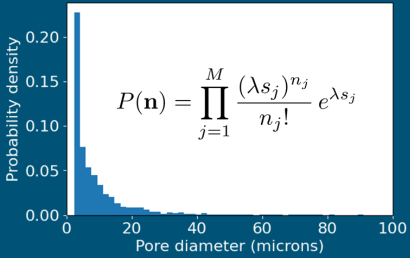
- Porosity data known [9, 10].
- Utilize calibrations *sans* damage [9].

Geometry & mesh using `Cubit`.

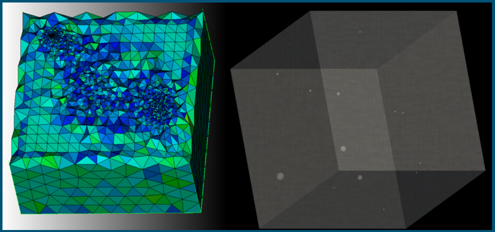
- Poisson point process for pore placement.
- Nominally 10 pores per 360 μm cube [11].
- Software from `structMechTools`.

FEM calculations using `Sierra` [12].

- Randomly sampled deformations [13].
- 50 + 3 + 9 + 3 load cases on all 50 meshes.
- 800,000 to 3,600,000 composite tetrahedra.
- Von Mises yield, Voce hardening [9].



$$P(\mathbf{n}) = \prod_{j=1}^{M} \frac{(\lambda s_j)^{n_j}}{n_j!} e^{\lambda s_j}$$



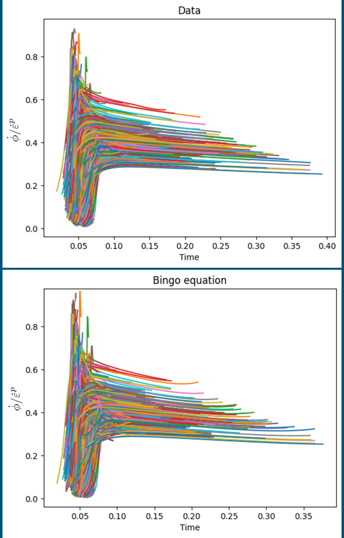
Damage

Explicit training data:

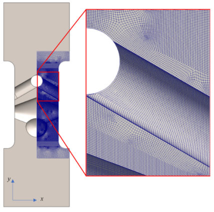
- $\dot{\phi}/\dot{\varepsilon}_p$, the scaled damage rate.
- $(1-\phi)$, with pore volume fraction ϕ.
- $T = \sigma_h/\sigma_e$, i.e., the triaxiality.
- The Lode factor $L = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3}$.
- Use volume averaging in results.

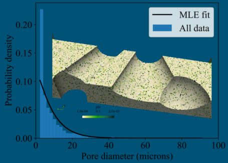
$$\dot{\phi} = \sqrt{\frac{3}{2}}\,\dot{\varepsilon}_p\,\frac{1-(1-\phi)^{n+1}}{(1-\phi)^n}\,\sinh\left[\frac{2(2n-1)}{2n+1}\,\frac{\sigma_h}{\sigma_e}\right]$$

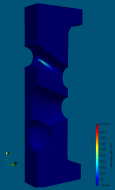
$$\dot{\phi}(\dot{\varepsilon}^p,\phi,T,L) = \dot{\varepsilon}^p*(1/0.222902)*(((-494.943443+(1-\phi)+(474.711387)*((1-\phi)))*((1-\phi)-(((1-\phi))*((1-\phi)))))*(((1/0.222902)*((1-\phi)-(((1-\phi))*((1-\phi))))-((1-\phi)))*(1/((1/((1/0.222902)*((1-\phi)-(((1-\phi))*((1-\phi))))-((1-\phi))))*(-240.262721+(241.293903)*(1/(1-\phi))+(-441.639817)*(((1-\phi)-(((1-\phi))*((1-\phi))))*(sinh(((1-\phi))*((1-\phi))))))+cosh(sinh(((1-\phi))*((1/T)*(cosh((1-\phi)))))))))$$

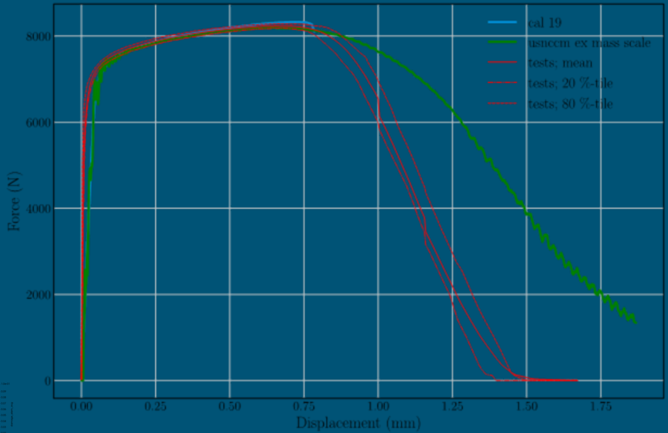


Damage




MLE fit
All data
Probability density
Pore diameter (microns)
0.00
0.05
0.10
0.15
0.20
0
20
40
60
80
100




cal 19
usnccm ex mass scale
tests; mean
tests; 20 %-tile
tests; 80 %-tile
Force (N)
Displacement (mm)
0
2000
4000
6000
8000
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75

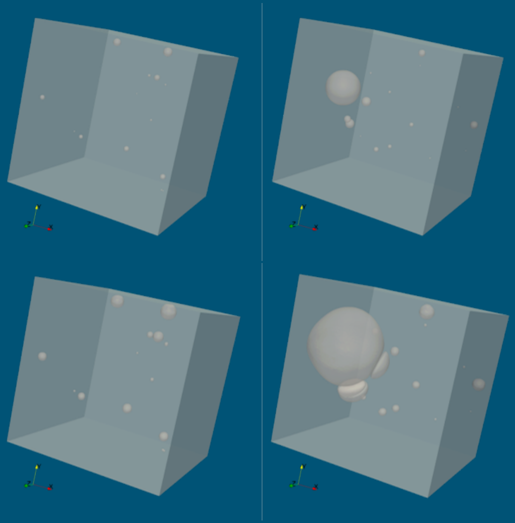
Damage

GPSR for damage evolution:

- Void growth is captured nicely.
- Void coalescence is missing.
- Overall, good results so far.

Many considerations going forward:

- Integrated fitness of damage evolution.
- Choice of applied loads and paths.
- Void coalescence without damage.
- Could void nucleation be included?
- Non-triaxial loads more influential?
- Grains morphologies, eventually.
- Spatially-varying damage models?



Conclusion

GPSR for solid mechanics:

- Genetic combinations of mathematical expressions fit to data.
- Training data generated by microstructural FEM calculations.
- New analytic relations unattainable through traditional derivation.

Repeating the process in search of a better damage law:

- Generating useful data and using it wisely is challenging in general.
- Refining microstructural features, focusing on coalescence.
- Revisiting applied load cases and judgement of fitness.

Other ongoing work and preliminary ideas:

- GPSR for stress intensity factor and fatigue predictions.
- Other microstructural problems, like Mullins & Payne effects in filled rubbers.
- Could existing constitutive models be genetically combined?

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